
Review for the CFA® Exams

# Lecture Notes

## Quantitative Methods 1
## Study Session 2

**Level I**

2009 Edition

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# Quantitative Methods 1

Level 1
Study Session 2

# I. Time Value of Money

Core Concepts:

A. Solve time value of money questions involving present value and future value of lump sums and annuities, assuming various compounding periods.

B. Distinguish between a stated annual rate and an effective annual rate and convert one to the other.

**Example:** Tom invests £10,000 and leaves it invested for 5 years. At the end of 5 years the account is worth £17,000. What rate of return was earned each year?

**Solution:**

| | | |
|---|---|---|
| PV | = | 10,000 |
| FV | = | -17,000 |
| $n_p$ | = | 5 |
| $i_p$ | = | ? |

**HP12C:** 10,000 [PV] 17,000 [CHS] [FV] 5 [n] [i] = 11.20%

**TIBA2+:** 10,000 [PV] 17,000 [+/-] [FV] 5 [N] [CPT] [I/Y] = 11.20%

## Compounding Periods and Interest Rates

- Increasing the frequency of compounding increases the total ending value.

- **Example:** Jane invests €100,000 in a certificate of deposit earning 4% per year. What is the ending value of the account 3 years later if the rate is compounded annually, quarterly, monthly, or daily?

| | PV | $n_p$ | $i_p$ | FV= |
|---|---|---|---|---|
| Annually: | 100,000 | 3x1=3 | 4/1=4 | 112,486.40 |
| Quarterly: | 100,000 | 3x4=12 | 4/4=1 | 112,682.50 |
| Monthly: | 100,000 | 3x12=36 | 4/12=.3333 | 112,727.19 |
| Daily: | 100,000 | 3x365=1,095 | 4/365=.01096 | 112,748.94 |

**Quantitative Methods 1**
**Time Value of Money** Question 1

You have just inherited $50,000 and plan to deposit it immediately in a bank account that pays 6% interest. No other deposits or withdrawals are made. In two years, what would be the account balance assuming monthly compounding, and would the balance be lower or higher assuming quarterly compounding?

| | *Monthly* | *Quarterly* |
|---|---|---|
| a. | $56,400 | Lower than monthly |
| b. | $53,100 | Lower than monthly |
| c. | $56,400 | Higher than monthly |

**Quantitative Methods 1**
**Time Value of Money** Question 2

Two single lump-sum investments have the following characteristics:

| | Compounding Periods | Stated Interest Rate |
|---|---|---|
| Investment A | Fewer | Lower |
| Investment B | More | Higher |

If both investments have the same future value, which should have the lowest cost?

a. Investment A.
b. Investment B.
c. They will cost the same.

**Quantitative Methods 1**
**Time Value of Money**

**Example:** Sue invests $5,000 in a 24-month deposit. At maturity it is worth $6,000. What is the rate of return, expressed as an SAIR and EAR? Which of the two is the "higher" rate of return?

Solution: PV = 5,000
FV = -6,000
$n_P$ = 24
$i_P$ = ?

**HP12C:** 5000 [PV] 6000 [CHS] [FV] 24 [n] [i] = .76257

**TIBA2+:** 5000 [PV] 6000 [+/-] [FV] 24 [N] [CPT] [I/Y] = .76257

SAIR = $i_P$ x m = .76257 x 12 = 9.15%
EAR = $(1+i_P)^m - 1 = (1 + .0076257)^{12} - 1 = .0955 = 9.55\%$

While the EAR is the "higher" number, in this question a 9.15% SAIR and 9.55% EAR generate the same dollar return of $1,000 on the initial $5,000 invested.

**Quantitative Methods 1**
**Time Value of Money** Question 3

If an investment has an APR of 18% and is compounded quarterly, its effective annual rate (EAR) is *closest* to:

a. 18.00%
b. 18.81%
c. 19.25%

# Annuities

- **Ordinary Annuity:** a time value of money problem in which an equal cash flow is added (or subtracted) at the ***end*** of each subperiod.

- **Annuity Due:** a time value of money problem in which an equal cash flow is added (or subtracted) at the ***beginning*** of each subperiod.

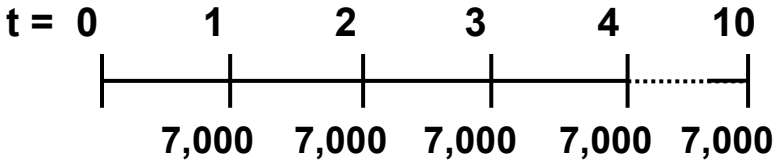
- **Payment (PMT):** the equal cash flow that occurs each subperiod in an annuity.

  Hint: Always assume an annuity is an ordinary annuity unless the facts clearly indicate otherwise.

**Quantitative Methods 1**
**Time Value of Money**

**Example:** Bob adds $7,000 per year to an account for 10 years. He earns 6% per year on the money. What is the account worth at the end of the 10 years?

**Solution:**



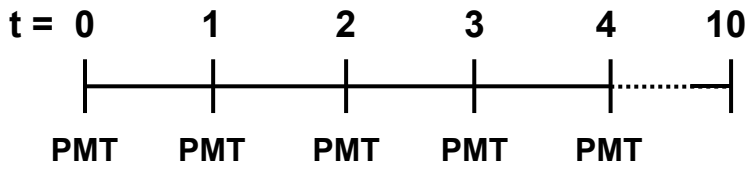

PMT =
N =
$i_P$ =

FV = ?

**Quantitative Methods 1**
**Time Value of Money**

**Example:** Bob wants to deposit an equal amount each year into a savings account, starting today. The annuity will earn 6% per year. At the end of 10 years he wants the account to be worth $112,000. How much must he deposit each year?

**Solution:**




**[Begin Mode]**

FV =
N =
$i_P$ =

PMT = ?

**Quantitative Methods 1**
**Time Value of Money** Question 4

An annuity requires 48 monthly payments of $500 beginning immediately and will have an ending value of $29,000 at maturity. The stated annual interest rate is *closest* to:

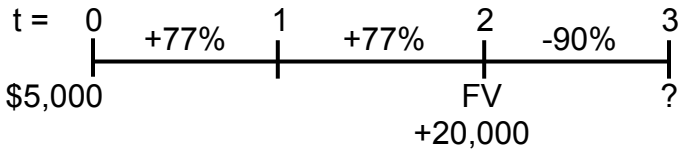
a. 9.00%
b. 9.50%
c. 10.00%

**Example:** Frank invests $5,000 into stock priced at $50 per share. The stock appreciates 77% per year for 2 years. Frank then invests another $20,000 in the stock and it depreciates 90%. What is the ending value of the portfolio?

**Solution:** There are several ways to solve this problem. One method is:

Step 1: Value at the end of 2 years:

PV = 5,000
$n_p$ = 2
$i_p$ = 77%
FV =




Step 2: Total value after the 90% depreciation:

PV =
$n_p$ = 1
$i_p$ = -90%
FV =

**Quantitative Methods 1**
**Time Value of Money** Question 5

A brokerage account has been set up with an initial investment of $50,000. This $50,000 portion will earn 6% annually. Quarterly investment amounts of $7,000 for 60 periods follow the initial investment, which will earn 7%. At the end of the 60 periods, $100,000 will be withdrawn. The balance remaining after the $100,000 withdrawal is invested in the market for three years. At the end of the three additional years, the account is closed and the final balance of $679,313.65 is withdrawn. What annual return was earned for the last 3 years of this investment?

a. –3.46%
b. –3.36%
c. +0.46%

# II. Discounted Cash Flow Applications

Core Concepts:

A. Calculate investment returns.

# Measuring Portfolio Returns

- Holding Period Return

$$HPR = \left[\frac{P_1 - P_0 + D_1}{P_0}\right] = \left[\frac{P_1 + D_1}{P_0}\right] - 1 = \frac{V_{End}}{V_{Begin}} - 1$$

- Money-weighted returns = IRR
  - Affected by client's decisions to contribute or withdraw funds.
- Time-weighted returns = geometric average
  - Unaffected by client's decisions

**Quantitative Methods 1**
**Discounted Cash Flow Applications** Question 6

An investor started the year with a $10,000 portfolio. He made a $1,000 contribution at the end of the first quarter, a $2,000 withdrawal at the end of the third quarter, and ended the year with a portfolio value of $10,553. The quarterly holding period returns for the investor's portfolio are as follows.

| **Q1** | **Q2** | **Q3** | **Q4** |
|---|---|---|---|
| 3% | -5% | 8% | 10% |

The money-weighted and time-weighted returns for the year are *closest* to:

| | *Money-Weighted* | *Time-Weighted* |
|---|---|---|
| a. | 14.34% | 3.84% |
| b. | 14.34% | 16.25% |
| c. | 3.59% | 16.25% |

# Money Market Yields

- Bank Discount Basis:

$$r_{BD} = \frac{F - P_0}{F}\left(\frac{360}{t}\right)$$

- Effective Annual Yield:

$$EAY = (1 + HPY)^{365/t} - 1$$

- Money Market Yield:

$$r_{MM} = HPY\left(\frac{360}{t}\right)$$

**Quantitative Methods 1**
**Discounted Cash Flow Applications** Question 7

A 120-day Treasury bill has a money market yield of 3.25%. Its holding period and effective annual yields are *closest* to:

| | *HPY* | *EAY* |
|---|---|---|
| a. | 1.06% | 3.32% |
| b. | 1.08% | 3.28% |
| c. | 1.08% | 3.32% |

# III. Statistical Concepts

Core Concepts:

A. Calculate and apply measures of central tendencies and dispersion for populations and samples.

**Quantitative Methods 1**
**Statistical Concepts**

**Population (N):** All of the items in a group. For example, the P/Es of all 500 stocks in the S&P 500.

**Sample (n):** A sample of some of the items in a group. For example, the P/Es of 50 stocks from the S&P 500.

**Arithmetic Mean:** A simple average of a population ($\mu_X$) or of a sample $\left(\bar{X}\right)$.

**Median:** The middle item from a group of data that has been arrayed from highest to lowest.

**Mode:** The item that appears most frequently in a group of data.

**Example:** Calculate the arithmetic mean, median, and mode of the following 4 years of portfolio returns.

| Year 1 | Year 2 | Year 3 | Year 4 |
|---|---|---|---|
| 12% | -5% | 12% | 3% |

## Solution:

*Arithmetic Mean:* (12% - 5% + 12% + 3%)/4 = 5.5%

*Median:* Array the data from high to low and select the middle value:

12%, 12%, 3%, -5%

- Because there are an even number of items, take the average of the 2 middle items: (12% + 3%)/2 = 7.5%

*Mode:* Select the item that appears most frequently: 12%

**Quantitative Methods 1**
**Statistical Concepts**

**Geometric Mean:** When a final ending value depends on a cumulative sequence of rates of return, the geometric mean is the single growth rate that produces the same ending value.

**Example:** A company's earnings increase by the following percentages each year. What is the geometric mean of its annual earnings growth rates?

| Year 1 | Year 2 | Year 3 | Year 4 |
|---|---|---|---|
| 12% | -5% | 12% | 3% |

**Quantitative Methods 1**
**Statistical Concepts**

**Variance:** The variance of a population ($\sigma^2$) is the average of the squared differences of each item in the population versus the population average.

$$\sigma_X^2 = \frac{\sum_{i=1}^{N}(X_i - \mu_X)^2}{N}$$

The variance of a sample ( $S_X^2$ ) is computed the same way except to take the average, divide by n - 1 instead of by N.

$$S_X^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

**Standard Deviation:** The standard deviation of both a population ($\sigma_X$) and a sample ($S_X$) are the square root of their variances.

**Quantitative Methods 1**
**Statistical Concepts** Question 8

Given the following sample of inventory ratios:

6.0, 11.2, 4.0, 8.8, 7.0

The sample standard deviation is *closest* to:

a. 2.45
b. 2.74
c. 3.45

**Coefficient of Variation:** This is a measure of *relative* dispersion as a percentage of the arithmetic mean.

$$CV = \frac{\sigma_X}{\mu_X}$$

**Sharpe Ratio:** This is a measure of return per unit of risk assumed. It is the excess return of a portfolio or asset above the risk-free rate per unit of total portfolio risk.

$$\text{Sharpe Ratio} = \frac{\overline{R_P} - \overline{R_F}}{\sigma_P}$$

**Quantitative Methods 1**
**Statistical Concepts** Question 9

The following table provides average return and variance information about three portfolio managers – Bob, Mark, and Rick:

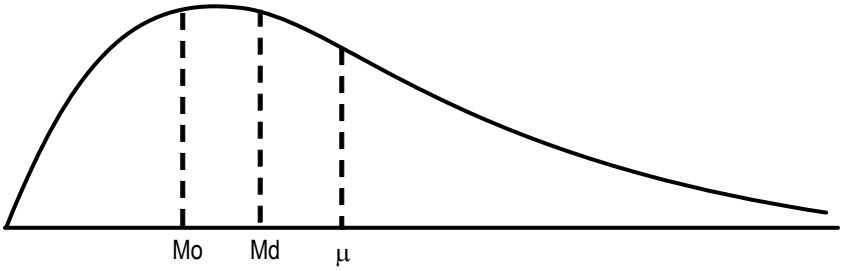
| | Bob Bobarino | Mark O. Witz | Rick N. Return |
|---|---|---|---|
| **Average Return (%)** | 15 | 13 | 9 |
| **Variance** | 81 | 49 | 36 |

Which of these managers has the best risk-adjusted return, as measured by the Sharpe Ratio, if the risk-free rate is 4%?

a. Bob.
b. Mark.
c. Rick.

# Skew

- Skewed distributions are not symmetrical
- Skew is caused by extreme values in the data, called outliers
- Outliers tend to "pull" the mean in the direction of the extreme value



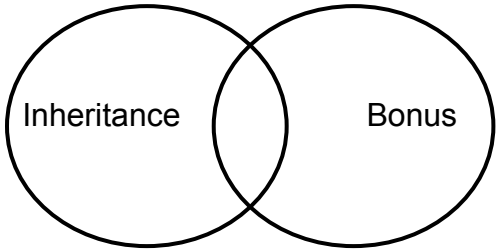
# IV. Probability Concepts

Core Concepts:

A. Apply probability rules and concepts to answer investment questions.

# The Addition Rule of Probability

**Example:** There is a 10% probability that you will receive a $100,000 bonus this year. There is a 1% probability you will inherit $100,000 this year. Finally, there is a 0.1% chance both events will occur this year. What is the chance you will receive at least $100,000?




P(A or B) = P($100,000 OR $200,000)
= P(A) + P(B) - P(A and B)
= 10% + 1% - 0.1% = 10.9%

# Multiplication Rule of Probability

**Example:** There is a 20% chance the economy will be in recession next year. If there is a recession, there is a 30% chance Quick Fly Airlines will declare bankruptcy. What is the chance there will both be a recession and Quick Fly will go bankrupt?

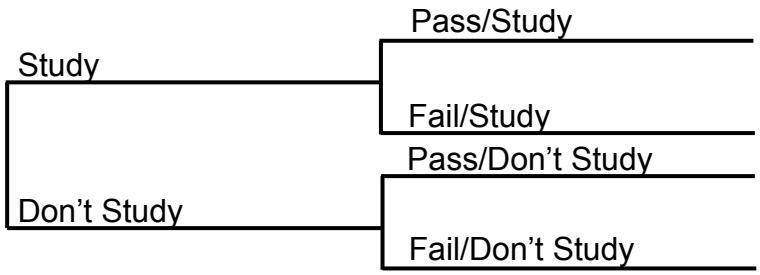
## Solution:

$$P(A \text{ and } B) = P(A)P(B\backslash A) = P(\text{Recession}) \times P(\text{Bankrupt}\backslash\text{Recession})$$

$$= 20\% \times 30\% = .20 \times .30 = 6\%$$

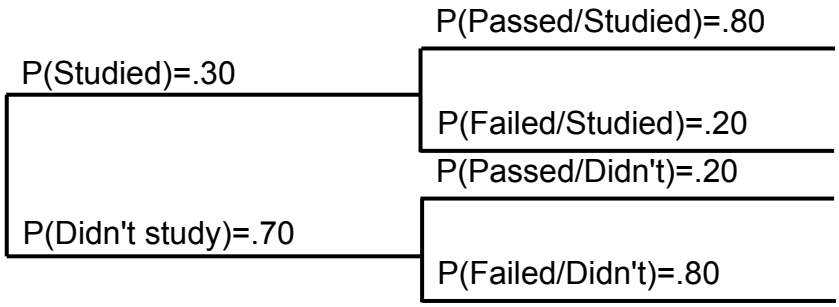
# Event Diagrams

- Event diagrams are used with the addition and/or multiplication rules to solve investment related questions.
- **Example:** Assume that 30% of CFA candidates study and prepare thoroughly for the exam. Those candidates have an 80% chance of passing the exam. Assume the remaining candidates have only a 20% chance of passing the exam. Draw an event diagram to analyze the probabilities of passing or failing the exam.



**Quantitative Methods 1**
**Probability Concepts**

# Analyzing the Event Diagram




P(Studied and Passed) = P(Studied) x P(Pass\Studied)

P(Passed) = P[(Passed\Studied) or (Passed\Didn't Study)]

**Quantitative Methods 1**
**Probability Concepts** Question 10

Information regarding the potential dividends per share (DPS) of The Cash Cow Company is as follows:

| DPS (X) | Probability of DPS P(X) |
|---|---|
| $0.70 | 0.15 |
| 0.95 | 0.20 |
| 1.25 | 0.35 |
| 1.40 | 0.20 |
| 1.80 | 0.10 |

What is the probability that DPS will be less than or equal to $1.25?

a. 0.30
b. 0.35
c. 0.70

**Quantitative Methods 1**
**Probability Concepts** Question 11

An analyst wants to forecast ROE for XYZ Company. She has assigned probability weights to the potential sales outcomes in this future period for which the forecast is relevant. The probability that sales will be good is 70% while the probability the sales will be poor is 30%. Within each scenario the probabilities are given for different ROE outcomes.

| Good Sales | Probability | ROE |
|---|---|---|
| 70% | 60% | 25% |
| | 40% | 17% |

| Poor Sales | Probability | ROE |
|---|---|---|
| 30% | 20% | 17% |
| | 80% | 4% |

The expected ROE of XYZ Company is *closest* to:

a. 15.33%
b. 15.75%
c. 17.24%

**Quantitative Methods 1**
**Probability Concepts** Question 12

Suppose that demand for automobiles increases, on average, in 3 out of every 5 years (60% probability). When the demand for automobiles increases, there is an 80% probability that the demand for steel will also increase. On the other hand, when the demand for automobiles decreases (40% probability), there is a 90% probability that the demand for steel will decrease. If industry analysts initially begin noticing that the demand for steel is increasing, but have no information about the demand for automobiles, then what is the posterior probability that the demand for automobiles is increasing?

a. 92.31%
b. 52.00%
c. 48.00%

**Quantitative Methods 1**
**Probability Concepts** Question 13

The following probability distribution (i.e., mutually exclusive and exhaustive collection of possibilities) for a company's possible profit margin for next year is as follows:

| Possible Profit Margin ($X_i$) | Probability of Profit Margin $P(X_i)$ |
|---|---|
| –7% | 0.10 |
| –3% | 0.50 |
| 4% | 0.40 |

What is the variance of this probability distribution?

a. 3.93
b. 15.44
c. 41.49

**Quantitative Methods 1**
**Probability Concepts**

- Expected portfolio return E(RP) is the weighted average return of the assets in the portfolio.

$$E(R_P) = w_1R_1 + w_2R_2 + \ldots + w_nR_n$$

- Portfolio variance is calculated as:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2COV_{1,2}$$

$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2r_{1,2}\sigma_1\sigma_2$$

- Portfolio standard deviation ($\sigma_P$) is the square root of portfolio variance.

# Covariance and Correlation

- **Covariance** ($COV_{X,Y}$) measures the way two variables change relative to each other.

$$COV_{XY} = r_{XY}\sigma_X\sigma_Y$$

- **Correlation** ($r_{X,Y}$) measures the degree to which two variables relate to each other.

$$r_{XY} = \frac{COV_{XY}}{\sigma_X\sigma_Y}$$

**Quantitative Methods 1**
**Probability Concepts** Question 14

Jones is considering a portfolio comprising two mutual funds. His associate presents him with the following covariance matrix.

**Covariance Matrix**

| Fund Symbol | VBIIX | FDVLX |
|---|---|---|
| VBIIX | 100 | -20 |
| FDVLX | -20 | 196 |

The correlation between the two funds is *closest* to:

a. 4.47
b. 0.51
c. -0.14

**Quantitative Methods 1**
**Probability Concepts** Question 15

The following table shows the individual weightings and expected returns for the three stocks in an investor's portfolio:

| Stock | Weight | $E(R_X)$ |
|---|---|---|
| V | 0.40 | 12% |
| M | 0.35 | 8 |
| S | 0.25 | 5 |

What is the expected return of this portfolio?

a. 8.33%
b. 8.85%
c. 9.05%

**Quantitative Methods 1**
**Probability Concepts** Question 16

Key information about a two-asset portfolio is presented below:

| Asset | Weight | Expected Return | Variance |
|---|---|---|---|
| J | 0.60 | 15% | 49 |
| C | 0.40 | 5 | 36 |

**$COV_{J,C}$ = 24**

**Correlation ($r_{JC}$) = 0.5714**

What is this portfolio's variance?

a. 34.92
b. 42.50
c. 43.80

**Quantitative Methods 2**

**Next Class:** Quantitative Methods 2

**Reading Assignment:** Study Session 3:

- Common Probability Distributions
- Sampling and Estimation
- Hypothesis Testing
- Technical Analysis

**NOTES**

## CLASS QUESTION SOLUTIONS

### Question 1 Solution

Choice "a" is correct. To compound monthly, remember to divide the interest rate by 12 (6%/12 = 0.50%) and the number of periods will be 2 years times 12 months (2 x 12 = 24 periods). The problem can be solved using the financial calculator:

HP (TI): 50,000 PV, 0.5 i (I/Y), 24 n (N), (CPT) FV = $56,357.99

Recall that the future value increase when the interest rate and/or number of compounding periods increases. Quarterly compounding has fewer periods than monthly, so the FV must be lower than monthly compounding.

### Question 2 Solution

Choice "b" is correct. Because the present value is a discounted future value, as the number of compounding periods and the interest rate are increased the present value becomes smaller. Picking some arbitrary numbers and computing the present values can prove this.

| | Compounding Periods | Annual Interest Rate |
|---|---|---|
| Investment A | 5 | 5% |
| Investment B | 10 | 9% |

Investment A HP (TI): FV 100, 5 i (I/Y), 5 n (N), (CPT) PV = 78.35

Investment B HP (TI): FV 100, 9 i (I/Y), 10 n (N), (CPT) PV = 42.24

### Question 3 Solution

Choice "c" is correct. Because this investment is compounded quarterly, we'll need to divide the APR by the appropriate number of compounding periods occurring in one year, which in this case is 4:

$$18/4 = 4.5$$

What this means, in effect, is that 4.5% return will be earned by the investment every 3 months (i.e., every quarter). Since this investment does not have to wait until year end to realize some portion of return, compounding creates "interest-on-interest" effects earlier, making the effective annual return greater than the stated APR. With 4 quarterly periods in one year, solve using the EAR formula:

$$i_{EAR} = (1 + i_p)^m - 1$$

$$i_{EAR} = (1 + .045)^4 - 1$$

$$= (1.045)^4 - 1$$

$$= 1.1925 - 1 = 0.1925, \text{ or } \underline{19.25\%} \text{ EAR}$$

Choice "a" is incorrect. This answer choice wrongly makes no distinction between APR and EAR.

Choice "b" is incorrect. This answer choice erroneously uses semiannual compounding.

**Question 4 Solution**

Choice "a" is correct. Because this is an annuity due (payments at the start of each period) the calculator must first be set to the beginning of period mode:

The HP calculator is placed in BEGIN mode by pressing g BEG.

The TI calculator is placed in BEGIN mode by pressing 2nd BGN, 2nd SET, 2nd QUIT

Using the financial functions, solve for the periodic interest rate:

HP (TI): 500 PMT, 48 n (N), 29,000 CHS (+/-) FV, (CPT) i (I/Y) = 0.75%

However, this percentage is a monthly rate because the time periods were entered as 48 months. It must be converted to a stated annual percentage (APR) by multiplying it by the number of compounding periods per year:

$$0.7532 \times 12 = \underline{9.04\%}$$

**HINT: Remember to reset the calculator back to the "normal" end mode convention.**

**Question 5 Solution**

Choice "b" is correct. This problem involves solving for different values, compounding periods, and rates of return - all occurring within the same span of time. As a result, the computation is trickier than most and must be done in a series of steps.

First, take the initial $50,000 portion and solve for its value at the end of 60 quarters, increasing at 6% annually. However, because the 6% growth rate is annual, the time periods must be annual as well. With 4 quarters in a year, there are 60 quarters in 15 years.

60/4 = 15 years

Now, solve for the future value of the $50,000 initial investment:

**HP12C:** 50,000 [CHS] [PV] 15 [n] 6 [i] [FV] FV = 119,827.91

**TIBA2+:** 50,000 [+/–] [PV] 15 [N] 6 [I/Y] [CPT] [FV] FV = 119,827.91

This portion will be worth $119,827.91 at the end of 15 years.

The second step of this problem calls for calculating the future value of an ordinary annuity (recurring, even payments occurring at the end of each period), in this case, an annuity of $7,000 payments occurring quarterly for 60 periods and earning 7%. Because no explicit direction is given as to the compounding frequency of 7%, the "normal" assumption is to treat its compounding frequency as corresponding to the time periods given in the question. Those are quarterly. Thus, the 7% interest rate should be converted to a periodic quarterly rate.

7/4 = 1.75

(continued)

Now, calculate the value of this annuity portion at the end of 60 periods:

**HP12C:** 7,000 [CHS] [PMT] 60 [n] 1.75 [i] [FV] FV = 732,756.51

**TIBA2+:** 7,000 [+/–] [PMT] 60 [N] 1.75 [I/Y] [CPT] [FV] FV = 732,756.51

The future value of the annuity is $732,726.51. Now aggregate this future value with the future value of the initial $50,000 investment from step one.

$732,726.51 + 119,827.91 = $852,554.42. This is the value of both portions 15 years (60 quarterly periods) from today, grown at their respective rates of return and compounding periods.

At this time, $100,000 will be withdrawn, reducing the balance to:

$852,554.42 - $100,000 = $752,554.42

Finally, the problem states the account will be worth $679,313.65 three years after that. In other words, 18 years after the initial deposit. What rate of return is earned during the last 3 years? To solve, treat the $752,554.42 as a present value that decreases to a future value of $679,313.65 over 3 years and solve for i:

**HP12C:** 752,554.42 [CHS] [PV] 3 [n] 679,313.65 [FV] [i] i = –3.36

**TIBA2+:** 752,554.42 [+/–] [PV] 3 [N] 679,313.65 [FV] [CPT] [I/Y] I/Y = –3.36

There was a -3.36% annual return for the last 3 years.

Choice "a" is incorrect. This is a filler answer that could potentially cause the candidate to second-guess himself with the negative sign and a value similar to the correct answer (choice "b").

Choice "c" is incorrect. This investment lost money during those last three years. Therefore, the return cannot be positive.

**Question 6 Solution**

Choice "b" is correct. The money-weighted return is simply the IRR. To calculate the *quarterly* IRR for the portfolio, use the cash flow functions of the financial calculator. Cash inflows are input as negative numbers and cash outflows are positive numbers. The value of the portfolio at the end of the year is considered a cash outflow because that is the amount you could potentially withdraw if you liquidated the portfolio.

| HP12C | TIBA2+ |
|---|---|
| 10,000 CHS g $CF_0$ | CF 10,000 +/– ENTER ↓ |
| 1,000 CHS g $CF_j$ | 1,000 +/– ENTER ↓ ↓ |
| 0 g $CF_j$ | 0 ENTER ↓ ↓ |
| 2,000 g $CF_j$ | 2,000 ENTER ↓ ↓ |
| 10,553 g $CF_j$ f IRR | 10,553 ENTER IRR CPT |

The answer is: 3.5856%.

This is the periodic IRR (quarterly). To annualize the return, multiply by 4 (quarters in a year).

$$APR_{M\text{-}W} = i_P m = 3.5856 \times 4 = 14.34\%$$

The time-weighted return is the geometrically linked subperiod returns.

$$R_{T\text{-}W} = (1 + r_{Q1})(1 + r_{Q2})(1 + r_{Q3})(1 + r_{Q4}) - 1 = (1.03)(.95)(1.08)(1.10) - 1 = 16.25\%$$

Note that, since the subperiods constitute one year, the annual time-weighted rate of return does not require the fourth root. The nth root would apply when averaging a series of *annual* holding period returns to compute the geometric or time-weighted average.

**Question 7 Solution**

Choice "c" is correct. This problem can be solved with just a little algebra.

$$r_{MM} = HPY\left(\frac{360}{t}\right) = HPY\left(\frac{360}{120}\right) = 3.25$$

$$HPY = 3.25\left(\frac{120}{360}\right) = \underline{\underline{1.08\%}}$$

$$EAY = (1 + HPY)^{365/t} - 1 = (1.0108)^{365/120} - 1 = \underline{\underline{3.32\%}}$$

Note that these are the same equations used for EAR and APR.

**Question 8 Solution**

Choice "b" is correct. First, find the variance of this sample. Since it's a sample (not a population), use the sample variance formula:

$$S_X^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$$

However, to calculate variance the arithmetic mean is required:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{6 + 11.2 + 4 + 8.8 + 7}{5} = \underline{7.40}$$

Now, figure out the sum of the squared differences for this sample:

| | | | | Squared difference |
|---|---|---|---|---|
| 6.00 | − | 7.40 | = −1.40 | 1.96 |
| 11.20 | − | 7.40 | = 3.80 | 14.44 |
| 4.00 | − | 7.40 | = −3.40 | 11.56 |
| 8.80 | − | 7.40 | = 1.40 | 1.96 |
| 7.00 | − | 7.40 | = −0.40 | 0.16 |
| | | | | 30.08 |

Then divide the sum of squared differences by the sample size "n" minus 1, for the sample variance of:

$$S_X^2 = \frac{30.08}{5-1} = \underline{7.52}$$

The final step is to take the square root of this variance:

$$Sx = \sqrt{S_X^2} = \sqrt{7.52} = \underline{\underline{2.74}}$$

The standard deviation of the inventory turnover ratios is 2.74.

Choice "a" is incorrect. This answer choice was derived from a flawed variance calculation where the sum of squared deviations was divided by "n," not "n - 1" as should be the case with sample variance and standard deviation calculations.

Choice "c" is incorrect. This is an arbitrary answer choice.

**Question 9 Solution**

Choice "b" is correct. This question is solved by calculating the Sharpe Ratio for each of the managers. Then select the manager with the highest ratio. The formula for the Sharpe Ratio is:

$$\text{Sharpe ratio} = \frac{\overline{R_P} - \overline{R_F}}{\sigma_P}$$

Because the standard deviation measures for the Sharpe Ratio denominators are not given, they must be found as the square root of the given variances:

| | **Bob Bobarino** | **Mark O. Witz** | **Rick N. Return** |
|---|---|---|---|
| **Variance** | 81 | 49 | 36 |
| **Standard Deviation** | $\sqrt{81} = 9$ | $\sqrt{49} = 7$ | $\sqrt{36} = 6$ |

Armed with the standard deviations of each manager's portfolio, calculate the Sharpe Ratio for each manager (bearing in mind that the risk-free rate is 4%):

| **Bob Bobarino** | **Mark O. Witz** | **Rick N. Return** |
|---|---|---|
| $\frac{15-4}{9} = 1.222$ | $\frac{13-4}{7} = 1.286$ | $\frac{9-4}{6} = 0.833$ |

After computing the Sharpe Ratio for each manager, Mark turns out to have the highest risk-adjusted return.

Choice "a" is incorrect. Bob has only the second highest Sharpe Ratio value of the three managers.

Choice "c" is incorrect. Rick actually has the lowest Sharpe Ratio value of the three managers.

**Question 10 Solution**

Choice "c" is correct. The question asks about the probability that DPS will be *less than or equal to $1.25*, meaning that the probability of the $1.25 itself must also be included in deriving total cumulative probability. Adding the probabilities of the DPS values equal to or less than $1.25, we get:

Total Probability that DPS ≤ $1.25 = $0.15 + 0.20 + 0.35 = \underline{\underline{0.70}}$.

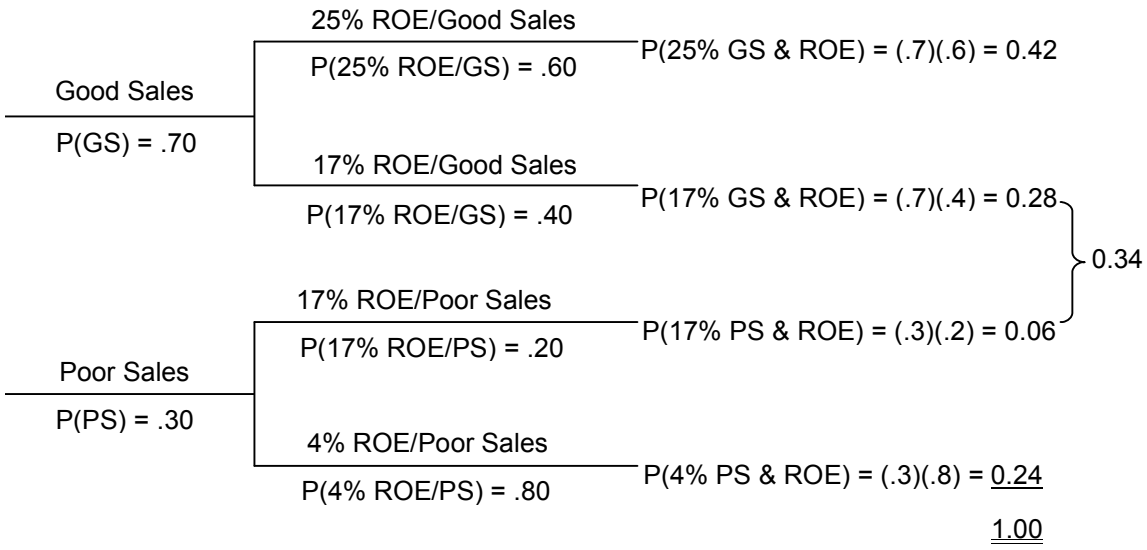
The probability that DPS will be less than or equal to $1.25 is 0.70, or 70%.

Choice "a" is incorrect. This is the probability that DPS will be greater than $1.25.

Choice "b" is incorrect. This is the probability that DPS will be less than $1.25.

**Question 11 Solution**

Choice "c" is correct. The event diagram outlining the analyst's forecast should resemble the one below:




From the event diagram, the probability of the company experiencing good sales AND a 25% ROE is 0.42. Also, the probability of the company experiencing 17% ROE is 0.34, regardless of the sales outcome. Finally, the probability of the company experiencing poor sales AND a 4% ROE is 0.24. Note that these entire probabilities sum up to 1.00, meaning that this event diagram encompasses the mutually exclusive and exhaustive scenarios outlined in the problem. We can now construct a probability-weighted mean to derive the expected ROE of the XYZ Company:

| **ROE** ($X_i$) | **Probability** $P(X_i)$ | $X_iP(X_i)$ |
|---|---|---|
| 25% | 0.42 | 10.50% |
| 17% | 0.34 | 5.78 |
| 4% | 0.24 | 0.96 |
| | 1.00 | 17.24% |

$$E(X) = \sum_{i=1}^{n} X_i P(X_i) = 17.24\%$$

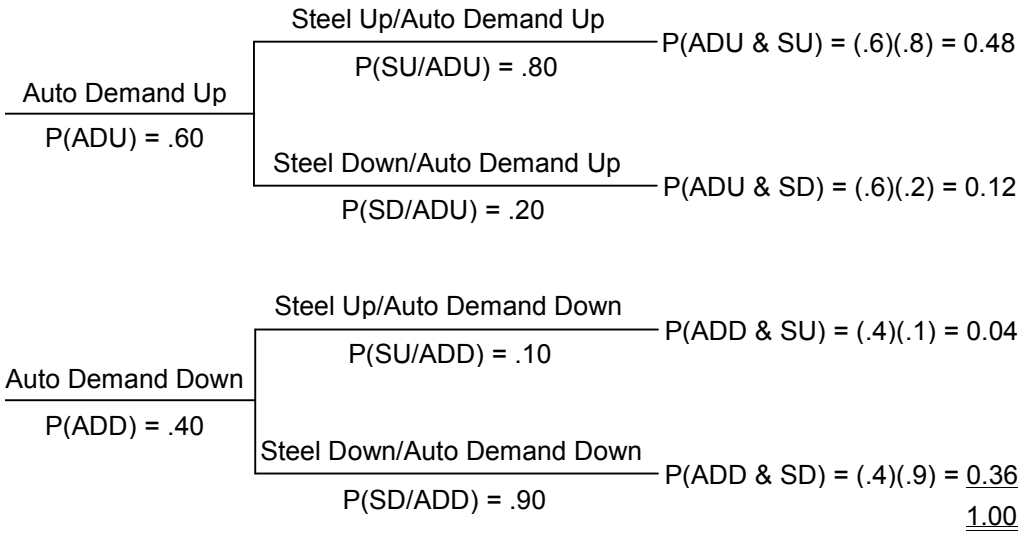
17.24% is XYZ's expected ROE.

Choice "a" is incorrect. This is simply the straight average of 25, 17, and 4.

Choice "b" is incorrect. This is simply the straight average of 25, 17, 17, and 4.

**Question 12 Solution**

Choice "a" is correct. This question calls for the candidate to use Bayes' Theorem to solve for posterior probability. First, set this question up by sketching an event diagram reflecting the mutually exclusive and exhaustive scenarios discussed in the question:




The total probability that the demand for steel will increase (from the total probability rule) is:

$$\begin{aligned} P(\text{Steel Up}) &= P(\text{Auto Demand Up \& Steel Up}) + P(\text{Auto Demand Down \& Steel Up}) \\ &= 0.48 + 0.04 = \underline{0.52} \end{aligned}$$

Given the information that the demand for steel is increasing, it can be deduced that the posterior probability that auto demand is also increasing is:

$$P(\text{Auto Demand Up/Steel Up}) = \frac{P(\text{Auto Demand Up \& Steel Up})}{P(\text{Steel Up})} = \frac{0.48}{0.52} = 0.9231, \text{or } \underline{\underline{92.31\%}}$$

The prior (unconditional) probability that the demand for autos would be increasing is 60%. However, given the new information that the demand for steel has, in fact, been increasing raises the probability to 92.31% that (as a result of this new information) demand for autos will increase.

Choice "b" is incorrect. This number is actually the probability that steel demand will be up, regardless of the economic condition. However, by itself, it tells us nothing about posterior probability.

Choice "c" is incorrect. This is the prior probability that steel will be up at the same time the demand for autos is up (from the event diagram). However, this selection does take into account that once the analysts get actual information about an increase in steel demand, the new information will "funnel back" to the original probability and alter it.

**Question 13 Solution**

Choice "b" is correct. As with any variance calculation, it is first necessary to calculate the arithmetic mean. For a probability distribution, this will be a probability weighted mean:

| Possible Profit Margin ($X_i$) | | Probability of Profit Margin $P(X_i)$ | |
|---|---|---|---|
| –7% | × | 0.10 | = –0.70% |
| –3% | × | 0.50 | = –1.50% |
| 4% | × | 0.40 | = 1.60% |
| | | | $\Sigma = E(X) = -0.60\%$ |

Then find the sum of the squared and weighted differences. This can be done as follows:

| $P(X_i)\,[X_i - E(X)]^2$ | |
|---|---|
| $.10[-7 - (-.6)]^2$ | = 4.096 |
| $.50[-3 - (-.6)]^2$ | = 2.880 |
| $.40[\ 4 - (-.6)]^2$ | = 8.464 |

$$\text{Variance} = \sum_{i=1}^{n}[X_i - E(X)]^2 P(X_i) = 15.440$$

Choice "a" is incorrect. This is actually the standard deviation of the company's profit margin probability distribution, but the question asks for the variance.

Choice "c" is incorrect. It is an arbitrary number.

**Question 14 Solution**

Choice "c" is correct. The covariance of an asset with itself is equal to its variance. Therefore, the variance of VBIIX is 100 (standard deviation = 10) and the variance of FDVLX is 196 (standard deviation = 14). Given the covariance between two assets in the matrix (-20), their correlation can be computed using the following formula:

$$r_{V,F} = \frac{COV_{V,F}}{\sigma_V \sigma_F} = \frac{-20}{(10)(14)} = \frac{-20}{140} = -0.14$$

**Question 15 Solution**

Choice "b" is correct. To solve this problem, we need to use the formula for the expected return of a portfolio:

$$E(R_P) = w_1E(R_1) + w_2E(R_2) + \ldots + w_nE(R_n)$$

Multiplying the weight of each asset by the expected return of each asset, then summing up, produces:

$$E(R_P) = 0.40(12) + 0.35(8) + 0.25(5) = \underline{8.85\%}$$

8.85% is the expected return of this portfolio.

Choice "a" is incorrect. This answer choice is actually the straight arithmetic average of all three returns, which is wrong because it doesn't factor the probability weightings into the calculation.

Choice "c" is incorrect. This is an arbitrary answer choice.

**Question 16 Solution**

Choice "a" is correct. The variance of the portfolio can be computed using either of the following formulas:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2COV_{1,2}$$

or, alternatively,

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2r_{1,2}\sigma_1\sigma_2$$

Plugging all of the factors into the first formula:

$$\sigma_P^2 = (0.60)^2 49 + (0.40)^2 36 + 2(0.60)(0.40)24 = \underline{\underline{34.92}}$$

To solve this problem using the second equation, first calculate the standard deviations for assets J & C:

$$\sigma_J = \sqrt{49} = 7$$
$$\sigma_C = \sqrt{36} = 6$$

With this, the same portfolio variance, 34.92, can be derived from the second formula as well:

$$\sigma_P^2 = (0.60)^2 49 + (0.40)^2 36 + 2(0.60)(0.40)0.5714(7)(6) = \underline{\underline{34.92}}$$

Choice "b" is incorrect. This is merely an arithmetic average of the variances of the two individual assets, which is not the correct mathematical way to obtain portfolio variance.

Choice "c" is incorrect. This is merely a weighted average of the variances of the two individual assets, which is not the correct mathematical way to obtain portfolio variance.